INFLUENCE OF HALL CURRENTS ON THE ACCELERATION OF A CONDUCTING GAS IN ITS OWN MAGNETIC FIELD

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In [1] the author gave a solution of the problem of how the Hall currents influence the flow pattern of a conducting gas which is accelerated in a channel to high velocities in external electric and magnetic fields. The present article considers the influence of Hall currents on the acceleration of a plasma in its own magnetic field, i.e., the magnetic field induced by the currents flowing in the plasma.

1. We consider the flow of a conducting gas in a channel with constant cross section and rectilinear axis. We introduce a rectangular system of coordinates, directing the x axis along the axis of the channel, and placing the origin on the lower wall in the initial cross section of the channel. We direct the y axis at right angles to the lower wall, and by y_0 we denote the constant height of the channel. We formulate the assumptions on which the problem will be solved.

(1) All the required quantities are independent of the z coordinate.

(2) The component of the electric field strength vector $\mathbf{E}$ in the direction of the z axis is equal to zero. The potential difference $\varphi(x)$ applied to the upper and lower walls of the channel is such that $E_x \ll E_y = E(x, y)$.

(3) There are no external magnetic fields.

(4) The plasma is neutral on the average. The conductivity σ = const and the parameter $\omega\tau = k$ = const (ω is the cyclotron frequency of the electrons and τ is the time between collisions). The quantity k is such that k^2 may be neglected in comparison with unity.

(5) The height y_0 of the channel is constant.

(6) Distortion of the electric and magnetic fields at the side walls of the channel and at the ends of the channel may be neglected.

(7) The gas is accelerated to high velocities ($\sim 10^7$ cm/sec), and the pressure gradient may be neglected in comparison with the electrodynamic force.

Assumptions (1), (3), (6) correspond to the case of flow in a cylindrical channel with coaxial electrodes in which the distance y_0 between electrodes is small compared with the radii of the electrodes. See [2], for example, as regards the acceptability of assumption (7).

The equations of continuity and motion and the generalized Ohm's law have the following form:

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0, \tag{1.1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{1}{4\pi} H \frac{\partial H}{\partial x},$$

$$\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{1}{4\pi} H \frac{\partial H}{\partial y}, \tag{1.2}$$

$$\frac{\partial H}{\partial y} = \frac{4\pi\sigma}{c(1+k^2)}\left[\frac{1}{c} vH - k\left(E - \frac{1}{c} uH\right)\right],$$
$$\frac{\partial H}{\partial x} = -\frac{4\pi\sigma}{c(1+k^2)}\left[E - \frac{1}{c} uH + k\frac{1}{c} vH\right]. \tag{1.3}$$

Here ρ is the density, u and v are the components of the velocity on the x and y axes, H is the component of the magnetic field strength on the z axis. Here the well-known equation of electrodynamics rot $\mathbf{H}$ = $= (4\pi/c)\mathbf{j}$ is taken into consideration.

The boundary conditions for u, v and ρ will be

$$u = u_{00}, \quad v = 0, \quad \rho = \rho_{00} \text{ for } x = 0;$$
$$v = 0 \text{ for } y = y_0 \text{ and } y = 0. \tag{1.4}$$

In addition to this we have the conditions for H and E

$$H = 0 \text{ for } x = L,\ y = 0; \qquad \int_0^{y_0} E\,dy = \varphi(x). \tag{1.5}$$

Clearly, the condition for E expresses the fact that the potential difference between the electrodes is a given quantity. As regards the condition for H, the following must be noted. If there were no Hall currents, then as a result of the fact that we neglect end effects, H should vanish at the end of the channel. The presence of currents flowing along the x axis may lead to the magnetic field strength being different from zero over the whole cross section at the end of the channel. However, as is well known, currents flowing along the surface of an axially-symmetric conductor give rise to a magnetic field outside the conductor only. Thus at the external electrode, i.e., at y = 0 the magnetic field strength remains equal to zero. At other points on the end section it must be determined from the solution of the problem.

Eliminating E from (1.3), we obtain

$$\partial H/\partial y - k\partial H/\partial x = (4\pi\sigma/c^2)\,vH \tag{1.6}$$

and for E itself we have from (1.3) the expression

$$E = (1/c)\,uH - (c/4\pi\sigma)(\partial H/\partial x + k\partial H/\partial y);$$

substituting in (1.5), we find

$$\int_0^{y_0}\left[\frac{1}{c} uH - \frac{c}{4\pi\sigma}\left(\frac{\partial H}{\partial x} + k\frac{\partial H}{\partial y}\right)\right] dy = \varphi(x). \tag{1.7}$$

Thus we must solve a system of four partial differential equations (1.1), (1.2), (1.6) for boundary conditions (1.4), (1.5) and (1.7). The unknown functions are ρ, u, v and H.

It was pointed out in [1] that in our case of acceleration to high velocities the influence of the Hall current leads to three different flow zones appearing in the channel. A zone of strong rarefaction (vacuum) is formed at the lower wall (anode) and there is a compression shock on the upper wall, while the core of the flow lies between these two zones. The ratio of the vacuum zone and the zone behind the compression shock to the channel height will be of the order of k, as was pointed out in [1].

First of all, we shall consider the core flow. We expand the required functions in series in powers of k

$$\rho = \sum_{i=0}^{\infty}\rho_i k^i, \quad u = \sum_{i=0}^{\infty} u_i k^i, \quad v = \sum_{i=0}^{\infty} v_i k^i, \quad H = \sum_{i=0}^{\infty} H_i k^i. \tag{1.8}$$

We solve the system of equations neglecting terms of order k^2. Setting expressions (1.8) in equations (1.1), (1.2) and (1.6) and equating coefficients of like powers of k, we obtain a system of equations for determining the zero-th and first approximations. We have for the quantities ρ_0, u_0, v_0, H_0 (noting that for $k = 0$ there are no Hall currents and the motion in the channel is one-dimensional)

$$v_0 = 0, \quad \frac{\partial(\rho_0 u_0)}{\partial x} = 0, \tag{1.9}$$
$$\rho_0 u_0 \frac{\partial u_0}{\partial x} = -\frac{1}{4\pi} H_0 \frac{\partial H_0}{\partial x}, \quad \frac{\partial H_0}{\partial y} = 0.$$

For ρ_1, u_1, v_1, H_1 we obtain

$$\frac{\partial(u_0\rho_1 + u_1\rho_0)}{\partial x} + \frac{\partial(\rho_0 v_1)}{\partial y} = 0$$
$$(u_0\rho_1 + u_1\rho_0)\frac{\partial u_0}{\partial x} + \rho_0 u_0 \frac{\partial u_1}{\partial x} = -\frac{1}{4\pi}\left(H_0 \frac{\partial H_1}{\partial x} + H_1 \frac{\partial H_0}{\partial x}\right) \tag{1.10}$$
$$\rho_0 u_0 \frac{\partial v_1}{\partial x} = -\frac{1}{4\pi} H_0 \frac{\partial H_1}{\partial y}, \quad \frac{\partial H_1}{\partial y} - \frac{\partial H_0}{\partial x} = \frac{4\pi\sigma}{c^2} v_1 H_0 .$$

Correspondingly, the boundary conditions for the zero-th approximation give

$$u_0 = u_{00}, \quad v_0 = 0, \quad \rho_0 = \rho_{00} \quad \text{for } x = 0; \quad H_0 = 0 \quad \text{for } x = L$$
$$y_0 [(1/c) u_0 H_0 - (c/4\pi\sigma) dH_0/dx] = \varphi(x). \tag{1.11}$$

In deriving the last relation from condition (1.7), we take into consideration the fact that the quantities of the zero-th approximation are independent of y and the corresponding expressions may thus be taken outside the integral sign.

From (1.4), (1.5), and (1.7) we also obtain the boundary conditions for the first approximation

$$u_1 = 0, \quad v_1 = 0, \quad \rho_1 = 0 \text{ for } x = 0; \quad H_1 = 0 \text{ for } x = L, y = 0$$
$$\int_0^{y_0} \left[\frac{1}{c}(u_0 H_1 + u_1 H_0) - \frac{c}{4\pi\sigma}\frac{\partial H_1}{\partial x}\right] dy = 0. \tag{1.12}$$

In the last condition of (1.12) the integral is taken over the whole height of the channel and not over the height of the core flow only, since the difference between these integrals will be of order k. And since the integral in (1.12) is itself the coefficient of a term of order k in the expansion of the expression which enters into (1.7), the error involved in changing the interval of integration will be of the order of k^2, i.e., of an order such as we everywhere neglect.

We consider the equations of the zero-th approximation. We have from the second and third equations of (1.9) and the boundary conditions

$$\rho_0 u_0 = \rho_{00} u_{00}, \tag{1.13}$$
$$\rho_{00} u_{00} u_0 + H_0^2/8\pi = \rho_{00} u_{00}^2 + H_{00}^2/8\pi .$$

Here H_{00} denotes the value of H_0 for $x = 0$, to be determined.

Expressing u_0 in terms of H_0 from (1.13) and setting this in the last relation of (1.11), we obtain an ordinary differential equation for H_0

$$\frac{c}{4\pi\sigma}\frac{dH_0}{dx} = \frac{H_0}{c}\left[u_{00} + \frac{1}{8\pi\rho_{00}u_{00}}(H_{00}^2 - H_0^2)\right] - \frac{\varphi}{y_0}. \tag{1.14}$$

An arbitrary constant appears when equation (1.14) is integrated. This may be expressed in terms of H_{00} from the condition that $H_0 = H_{00}$ for $x = 0$. Subsequently, H_{00} may be determined from the boundary condition (1.11). Thus, finding the quantities of the zero-th approximation reduces to the integration of a single ordinary differential equation of the first order.

The system of equations (1.10) for quantities of the first approximation is a system of four linear partial differential equations. With the known quantities of the zero-th approximation, its solution for boundary conditions (1.12) may be reduced to the calculation of quadratures. In fact, eliminating H_1 from the two last equations of (1.10), we obtain

$$\frac{\partial v_1}{\partial x} + \frac{\sigma H_0^2}{c^2 \rho_{00} u_{00}} v_1 + \frac{H_0}{4\pi\rho_{00}u_{00}}\frac{dH_0}{dx} = 0. \tag{1.15}$$

This is a linear equation in v_1, containing only one derivative of v_1 with respect to x. The coefficients of this equation are also functions of x only. Thus equation (1.15) may be integrated with respect to x. Its general solution will be

$$v_1 = e^{-\Lambda_0(x)}\left[f_1(y) - \frac{1}{4\pi\rho_{00}u_{00}}\int_0^x H_0 \frac{dH_0}{dx} e^{\Lambda_0(x)} dx\right],$$
$$\Lambda_0(x) = \frac{\sigma}{c^2\rho_{00}u_{00}}\int_0^x H_0^2 dx,$$

where $f_1(y)$ is an arbitrary function of y.

Substituting here the boundary condition (1.12), we get that $f_1(y) \equiv 0$ and

$$v_1 = -\frac{1}{4\pi\rho_{00}u_{00}} e^{-\Lambda_0(x)}\int_0^x H_0 \frac{dH_0}{dx} e^{\Lambda_0(x)} dx. \tag{1.16}$$

Thus the calculation of v_1 is reduced to quadratures. Next, from the last equation of (1.10), we find

$$H_1 = y\,[dH_0/dx + (4\pi\sigma/c^2) v_1 H_0] + f_2(x), \tag{1.17}$$

where $f_2(x)$ is an arbitrary function of x.

Since it follows from (1.16) that v_1 in the case under consideration depends on x only, from the first equation of (1.10) and the boundary conditions (1.12) we find

$$u_0\rho_1 + u_1\rho_0 = 0 . \tag{1.18}$$

Taking (1.18) and (1.13) into consideration, we may now write the remaining equation of (1.10) in the form

$$\rho_{00}u_{00}\frac{\partial u_1}{\partial x} = -\frac{1}{4\pi}\left(H_0\frac{\partial H_1}{\partial x} + H_1\frac{\partial H_0}{\partial x}\right).$$

Or integrating

$$\rho_{00}u_{00}u_1 + (1/4\pi) H_0 H_1 = f_4(y), \tag{1.19}$$

where $f_4(y)$ is an arbitrary function of y.

For $x = 0$ we have $u_1 = 0$ and, as follows from (1.17) and (1.12), H_1 equals

$$H_1(0, y) = y\,(dH_0/dx)_0 + f_2(0).$$

Taking this into account, we find that

$$f_4(y) = (1/4\pi) H_{00}\,[y\,(dH_0/dx)_0 + f_2(0)] .$$

Substituting the value obtained for $f_4(y)$ and also the value of H_1 from (1.17) into (1.19), we obtain an explicit expression for the dependence of u_1 on y:

$$u_1 = \frac{1}{4\pi\rho_{00}u_{00}} \times \tag{1.20}$$

$$\times \left\{ y \left[\left(\frac{dH_0}{\partial x} \right)_0 H_{00} - \frac{dH_0}{dx} H_0 - \frac{4\pi\sigma}{c^2} H_0{}^2 v_1 \right] + H_{00} f_2(0) - H_0 f_2(x) \right\}.$$

Thus H_1 and u_1 are expressed in terms of one arbitrary function of x: $f_2(x)$. This function must be determined from the last relation of (1.12). Setting the value of H_1 from (1.17) and u_1 from (1.20) into (1.12), we find the required equation for $f_2(x)$:

$$\int_0^{y_0} \left[2A(x)\, y + B(x) f_2(x) + C(x) f_2(0) - \frac{df_2}{dx} \right] dy = 0,$$

$$2A(x) = \frac{\sigma}{\rho_{00}u_{00}c^2} \left[\left(\frac{dH_0}{dx} \right)_0 H_{00} H_0 - \frac{dH_0}{dx} H_0{}^2 - \frac{4\pi\sigma}{c^2} H_0{}^3 v_1 \right] +$$

$$+ \frac{4\pi\sigma}{c^2} \frac{dH_0}{dx} u_0 + \left(\frac{4\pi\sigma}{c^2} \right)^2 H_0 u_0 v_1 - \left[\frac{d^2H_0}{dx^2} + \frac{4\pi\sigma}{c^2} \left(H_0 \frac{dv_1}{dx} + v_1 \frac{dH_0}{dx} \right) \right],$$

$$B(x) = \frac{4\pi\sigma}{c^2} \left(u_0 - \frac{1}{4\pi\rho_{00}u_{00}} H_0{}^2 \right), \qquad C(x) = \frac{\sigma}{\rho_{00}u_{00}c^2} H_{00} H_0 .$$

Here A(x), B(x) and C(x) are known functions of x. Integrating with respect to y and cutting the integration off at y_0, we obtain an ordinary linear differential equation for determining the unknown function $f_2(x)$:

$$df_2 / dx = B(x) f_2(x) + C(x) f_2(0) + y_0 A(x) .$$

Its general solution is

$$f_2(x) = \left[\exp\left(\int_0^x B dx \right) \right] \times$$

$$\times \left\{ f_2(0) + \int_0^x [C f_2(0) + y_0 A] \left[\exp\left(-\int_0^x B dx \right) \right] dx \right\}. \tag{1.21}$$

The constant $f_2(0)$ may be determined from the boundary condition (1.12) for H_1, which has not yet been used and which together with (1.17) shows that $f_2(L) = 0$. Thus Eqs. (1.16)–(1.18), (1.20), and (1.21) allow us to calculate all the required quantities of the first approximation by quadratures in the general case.

2. We shall complete all the calculations for the case when there is no Hall current and the function $\varphi(x)$ is given in optimal form, i.e., such that Joule losses are minimal for acceleration of the plasma in the channel to a given velocity. As was shown in [3], in this case the potential difference distribution along the channel axis should have the form

$$\varphi = (H_{00} y_0 / c)(1 - x/L)\{u_{00} + (H_{00}{}^2 / 8\pi\rho_{00}u_{00}) \times$$
$$\times [1 - (1 - x/L)^2]\} + cH_{00} y_0 / 4\pi\sigma L, \tag{2.1}$$

where H_{00} is the value of the magnetic field strength in the x = 0 cross section, determined from the expression

$$H_{00} = 4\pi\sigma\varphi_0 cL / y_0 (4\pi\sigma L u_{00} + c^2) \qquad (\varphi_0 = \varphi(0)) . \tag{2.2}$$

We introduce the dimensionless variables

$$x / L = x^*, \quad y / L = y^*, \quad \rho / \rho_{00} = \rho^*,$$
$${}_*H = {}^{00}H / H \,(4\pi\sigma L / c^2)\, u = u^*,$$
$$(4\pi\sigma L / c^2)\, v = v^*, \quad \sigma H_{00}{}^2 L / \rho_{00} u_{00} c^2 = \lambda . \tag{2.3}$$

We agree henceforth to omit the stars from the dimensionless variables.

For a potential difference distributed along the length of the channel according to the law (2.1), the zero-th approximation of the magnetic field strength in the channel and u_0 have the form

$$H_0 = 1 - x, \qquad u_0 = u_{00} + 0.5\lambda\, [1 - (1 - x)^2]. \tag{2.4}$$

From (1.16) we find

$$v_1 = \lambda e^{(\lambda/3)(1-x)^3} \int_0^x (1 - x)\, e^{-(\lambda/3)(1-x)^3} dx . \tag{2.5}$$

From (1.17) and (1.20) we obtain

$$H_1 = y\,(v_1 H_0 - 1) + f(x) \qquad (f(x) = f_2(x)/H_{00})$$
$$u_1 = -\lambda\, [y\,(1 - H_0 + v_1 H_0{}^2) + H_0 f - f_0] \qquad (f_0 = f(0)). \tag{2.6}$$

Setting the corresponding values of the quantities in expression (1.21) for $f_2(x)$ and reducing it to dimensionless form, we obtain

$$f(x) = \left[\exp\left(\int_0^x (u_0 - \lambda H_0{}^2)\, dx \right) \right] \times$$

$$\times \left[f_0 + \frac{1}{2} y_0 \int_0^x [2 y_0 \lambda H_0 f_0 - (u_0 + \lambda H_0) +\right.$$

$$\left. + v_1 (1 + u_0 H_0)] \exp\left(- \int_0^x (u_0 - \lambda H_0{}^2)\, dx \right) dx \right].$$

Taking (2.4) into consideration, we may calculate part of the integrals entering into this expression. Performing the corresponding calculations, we obtain

$$f(x) = e^{-H_0 u_0} \left\{ f_0 [e^{u_{00}} + J_1(x)] + \frac{1}{2} y_0 J_2(x) \right\},$$

$$J_1(x) = \int_0^x e^{H_0 u_0} H_0 dx, \tag{2.7}$$

$$J_2(x) = \int_0^x [v_1 (1 + u_0 H_0) - (u_0 + \lambda H_0)]\, e^{H_0 u_0} dx .$$

We determine the constant f_0. To do this we note that it follows from the boundary condition (1.12) for H_1 and (2.6) that we must have $f(1) = 0$. Making use of this, we easily calculate the last arbitrary constant f_0:

$$f_0 = -y_0 J_2(1) / 2\, (e^{u_{00}} + J_1(1)]. \tag{2.8}$$

Thus the required quantities in the flow core have the form

$$u = u_{00} + 0.5\lambda\, [1 - (1 - x)^2] - k\lambda\, \{y\, [1 - (1 - x)^2 +$$
$$+ \lambda\, (1 - x)^2 J(x)\, e^{(\lambda/3)(1-x)^3}] + (1 - x) f(x) - f_0\},$$
$$J(x) = \int_0^x (1 - x)\, e^{-(\lambda/3)(1-x)^3} dx \quad v = k\lambda J(x)\, e^{(\lambda/3)(1-x)^3}, \tag{2.9}$$
$$H = 1 - x + k\, \{f(x) + y\, [\lambda\, (1 - x)\, J(x)\, e^{(\lambda/3)(1-x)^3} - 1]\},$$

where $f(x)$ is given by (2.7), and f_0 by (2.8).

On the basis of the equations obtained we shall consider the flow picture in the core of the stream in more detail. From (2.5) we find

$$dv_1 / dx = \lambda (1 - x) [1 - \lambda (1 - x) e^{(\lambda/3)(1-x)^3} J (x)] .$$

Hence it is clear that the derivative dv_1/dx always vanishes at the end of the channel for x = 1. Thus, in the case under consideration, the velocity component on the y axis always reaches a maximum at the end of the channel, as distinct from the examples investigated in [1]. Values of v/k calculated from (2.5) for three values of λ (λ = 10 solid curve, λ = 3 broken curve, λ = 0.3 dot-dash curve) are shown in Fig. 1.

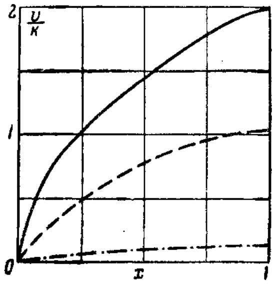

Fig. 1

The differential equation of the stream lines has the form

$$dy / dx = kv_1 / u_0$$

with an accuracy to small terms of higher order.

Integrating and using (2.4) and (2.5), we obtain the relation between y and x along a stream line

$$y = Y + k \int_0^x \frac{\lambda J (x) e^{(\lambda/3)(1-x)^3}}{u_{00} + 0.5\lambda [1 - (1 - x)^2]} dx, \qquad (2.10)$$

where Y is the value of y on the stream line for x = 0.

Stream lines calculated from this formula for u_{00} = = 3 and two values of λ are shown in Fig. 2 (λ = 10 solid curve and λ = 3 broken curve). It follows from (2.10) that with an accuracy to terms of order k^2 the different lines may be obtained one from the other by simply shifting along the y axis.

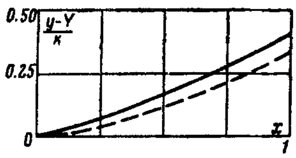

Fig. 2

3. In the case of acceleration by the magnetic field of the plasma itself, the flow behind the compression shock is calculated in exactly the same way as for an external magnetic field [1]. Estimates of the magnitude of the velocity v remain valid, and thus the expressions

$$\rho^1 / \rho^\circ = (\varkappa + 1) / (\varkappa - 1)$$
$$u_n^1 / u_n^\circ = (\varkappa - 1) / (\varkappa + 1), \qquad (3.1)$$

for density and velocity behind the shock will be valid in the first approximation as before.

Here $\varkappa$ is the ratio of specific heats at constant pressure and volume, the upper index ° designates quantities in the flow core in front of the shock, the upper index 1 designates quantities behind the shock, and u_n is the velocity component normal to the shock. If the gas is fully ionized, then we may assume that the quantity $\varkappa$ does not change on passing through the shock.

The expressions

$$u^1 = u_\tau^1 \cos\alpha + u_n^1 \sin\alpha, \quad v^1 = - u_\tau^1 \sin\alpha + u_n^1 \cos\alpha \qquad (3.2)$$

$$u_\tau = u \cos\alpha - v \sin\alpha, \quad u_n = u \sin\alpha + v \cos\alpha \qquad (3.3)$$

also remain valid.

Here α is the angle of inclination of the shock to the upper wall and u_τ is the velocity component tangential to the shock.

As shown in [1], we may assume that behind the shock $v \approx 0$ with the degree of accuracy adopted. Then taking into consideration that the tangential component of velocity does not change on passing through the shock, we obtain

$$\operatorname{tg}\alpha = u_n^1 / u_\tau^1 = u_n^1 / u_\tau^\circ = [(\varkappa - 1) / (\varkappa + 1)] u_n^\circ / u_\tau^\circ \qquad (3.4)$$

from (3.1) and (3.2).

Confining ourselves to terms of order k, it is possible to find from (3.3) and (3.4)

$$\operatorname{tg}\alpha = (\varkappa - 1) v^\circ / 2u^\circ, \qquad u^1 = u_\tau^1 = u_\tau^\circ = u^\circ, \qquad (3.5)$$

just as in the case of an external magnetic field.

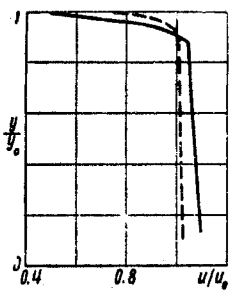

Fig. 3

The equations of continuity and motion behind the shock assume the form

$$\partial (\rho u) / \partial x = 0, \qquad \rho u \, \partial u / \partial x = - u_{00}\lambda H \partial H / \partial x$$

if basic terms only are retained and if we take into account that behind the shock $v \approx 0$.

Integrating these, we obtain

$$\rho u = \rho^1 u^1, \qquad \rho u^2 + 0.5 u_{00}\lambda H^2 = \rho^1 u^{12} + 0.5 u_{00}\lambda H^{12}, \qquad (3.6)$$

where ρ^1, u^1 and H^1 are the values of ρ, u and H on the stream line considered behind the shock at the place where it intersects the shock. Setting the expansions (1.8) in the first equation of (3.6) and taking (1.13), (1.18), (3.1), and (3.5) into consideration, we obtain

$$\rho u = [(\varkappa + 1) / (\varkappa - 1)] \rho_{00} u_{00} \qquad (3.7)$$

behind the shock.

We now note that with an accuracy to quantities of order k^2 we may assume that behind the shock the magnetic field strength H is independent of y. Actually, it follows from (1.3) that $\partial H / \partial y \sim kH / y_0$ in order of magnitude. Consequently, behind the shock we have

$$H(x, y) = H(x, y^1) + \int_{y^1}^{y} \frac{\partial H}{\partial y} dy = H(x, y^1) + O(k^2)$$

since the interval of integration $y - y^1 \sim ky_0$, where y^1 is the coordinate of the shock. Taking this into account, as well as (3.7) and (3.5), and also the fact that H does not suffer any discontinuity at the shock, we obtain from the second equation of (3.6) an expression for the velocity u behind the shock

$$u(x, y) = u[x^1(y), y] + [(\varkappa - 1)/2(\varkappa + 1)]\lambda\{H^2[x^1(y), y] - H^2[x, y^1(x)]\}. \tag{3.8}$$

Equation (3.8) gives an expression for the gas velocity behind the shock in terms of parameters in the flow core in front of the shock, in the channel cross section considered (the point x, $y^1(x)$) and in the cross section where the stream line intersects the shock (the point $x^1(y)$, y).

We obtain the relation between the coordinates of the shock by noting that $\operatorname{tg}\alpha = -dy^1/dx^1$. We substitute values of $\operatorname{tg}\alpha$ from (3.5) and, from (2.9), the quantities u and v [which enter into expression (3.5)], in the flow core and integrate to obtain

$$y^1 = y_0 - k\frac{\varkappa - 1}{2}\int_0^x \frac{v_1 dx}{u_{00} + 0.5\lambda[1 - (1 - x)^2]} \tag{3.9}$$

with an accuracy to quantities of order k^2.

Fig. 3 shows the velocity profiles at the exit from the channel calculated from (2.9) for the flow core and from (3.8) behind the shock. Along the abscissa axis is plotted the ratio of flow velocity to u_0, i.e., to the value of the velocity at the channel exit in the absence of Hall currents, and along the ordinate axis the value of y relative to the channel height (to y_0). The calculation has been carried out for $k = 0.2$, $y_0 = 0.3$, $u_{00} = 3$. The solid curve corresponds to $\lambda = 10$, the broken curve to $\lambda = 3$. The points of discontinuity on the curves correspond to the coordinate of the shock ($y^1/y_0 \approx 0.887$ for $\lambda = 10$ and $y^1/y_0 \approx 0.907$ for $\lambda = 3$). The lower points on the curves give the boundary of the vacuum region. For $\lambda = 10$ this boundary will be at $y/y_0 \approx 0.136$ and for $\lambda = 3$ at $y/y_0 \approx 0.114$.

It is clear from Fig. 3 that the general character of the influence exerted by Hall currents on the velocity profile for acceleration of a plasma in its own magnetic field remains the same as for acceleration in an external magnetic field [1], i.e., the velocity changes gently in the flow core and in the zone behind the shock in streams passing through the shock at the beginning of the channel, the velocity is 1.5–2 times less than in the main flow. There is a small difference in the fact that in the case where a gas is accelerated in its own magnetic field the velocity profile in the main flow is asymmetric with respect to the center line of the channel. This is associated with the fact that for self-field acceleration this field is asymmetric over the channel cross section.

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